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Stochastic Maintenance Optimization of Complex Industrial Systems under Uncertainty of Repair Time and Resources

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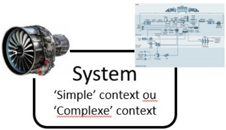
Contents

INTRODUCTION

Context : MPO Project

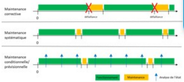
Technical and economic criteria for maintenance objectives

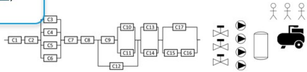


Collect and evaluate complex and heterogenous data for the maintenance

Methods and tools for diagnosis, prognosis (model-based and data-based)

The MPO project (Predictive Maintenance and Optimization)
IRT SystemX (https://www.irt-systemx.fr/en/projets/mpo/)
Over a period of 4 years (started in September 2018)





Optimizing maintenance scheduling (correctives and preventives) according to technical and economic criteria and execution results (analytic or simulation)

Consider different relevant maintenance according to results of monitoring, diagnosis and prognosis, and the technical and economic criteria

Maintenance Planning

A maintenance strategy determines the:

1. Components requiring preventive maintenance (PM).
2. Optimal frequency of PM actions.
3. Maintenance interval.
4. Failure risks.
5. Maintenance cost.

Assumptions

01 Multi-component systems

02 Corrective Maintenance (ABAO)
Preventive Maintenance (AGAN)

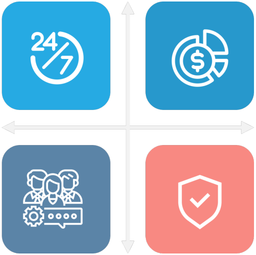
03 Resources +Repairmen

04 One Degradation Mode

⚠ However this may not always be the reality !

Maintenance Constraints

Availability, Reliability



Budget

Labor, Materials, Spares

Safety, Quality …etc.

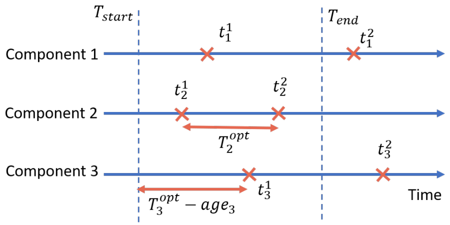
OPTIMAL INDIVIDUAL MAINTENANCE DATES

Optimal Individual Maintenance Dates

Individual maintenance dates

- We determine the individual maintenance date T_i^{opt}
- Scheduling maintenance tasks

$$t_i^1 = T_i^{opt} - age_i + T_{start} \tag{1}$$



PROBLEM FORMULATION

Problem identification

We consider:

- A system with n components

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- Each maintenance action has a $d_i > 0$ duration

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- A Planning horizon $[0, T]$

We define the following sets:

- $\mathcal{J}$: Planning Horizon Set $\mathcal{J} = 1, 2 \ldots, T$

Problem identification

We consider:

- A system with n components
- Each maintenance action has a $d_i > 0$ duration
- Each maintenance component requires $r_i > 0$ resources
- A Planning horizon $[0, T]$

We define the following sets:

- $\mathcal{J}$: Planning Horizon Set $\mathcal{J} = 1, 2 \ldots, T$
- $\mathcal{I}$: Maintenance components Set $\mathcal{I} = 1, 2, \ldots, n$

Parameters

- C_i^{prev} : component-specific PM action cost
- C_i^{correc} : component-specific CM action cost
- $\alpha_i,\ \beta_i$: Weibull Law shape and scale parameters for component i
- d_i : maintenance action duration of component i
- r_i : the number of resources required for the maintenance action of component i
- R : The total available resources in the system
- R_j : The total available resources at time j

Variables

1. Binary Variables:

$$m_{ij} = \begin{cases} 1, & \text{if object } i \text{ is maintained at instant } j \\ 0, & \text{otherwise} \end{cases}$$

2. Continuous Variables:
   - x_i : maintenance date of component i

Objective Function

- $cost(x_i)$ is the cost of maintenance of component i (at maintenance time x_i):

$$cost(x_i) = \frac{C_i^{prev} + C_i^{correc}\left(\frac{x_i}{\alpha_i}\right)^{\beta_i}}{x_i} \quad x_i \neq 0 \tag{2}$$

- Minimize the total cost of maintenance
(a non linear function):

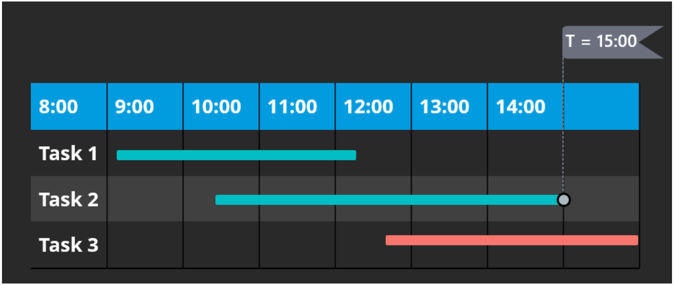
$$F(x) = \sum_{i=1}^{n} cost(x_i). \tag{3}$$

Constraints

Maintenance duration

- If a maintenance action for component i starts at j, it must be executed within the planning horizon.

$$jm_{ij} + (d_i - 1) \leq T \quad \forall i \in \mathcal{I} \quad \forall j \in \mathcal{T} \tag{4}$$



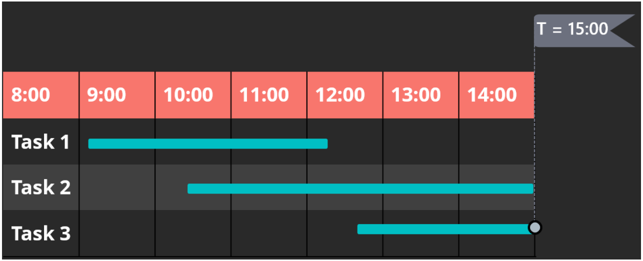
Constraints

Maintenance frequency

- Each component must be maintained once.

$$\sum_{j=1}^{T} m_{ij} \geq 1 \quad \forall i \in \mathcal{I} \tag{5}$$

$$m_{ij'} \leq 1 - m_{ij} \quad \forall j' > j \in \mathcal{T} \quad \forall i \in \mathcal{I} \tag{6}$$

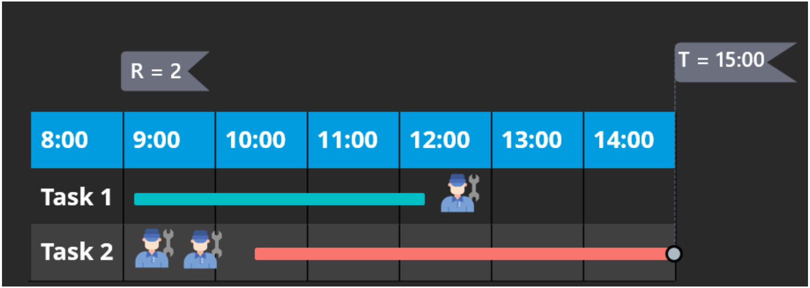


Constraints

Resource constraints

- A component can be maintained if and only if there are available resources.

$$\sum_{i=1}^{n} m_{ij} r_i \leq R_j, \forall j \in \mathcal{T} \quad (7)$$

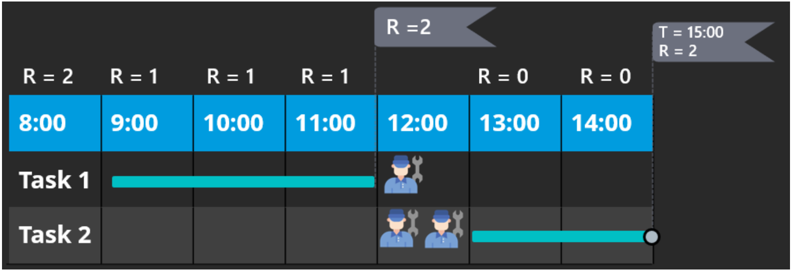


Constraints

Resources release constraints:

- Used resources are restored.
- PM actions can be executed simultaneously.

$$m_{ij} r_i \leq R - \sum_{\substack{k=0 \\ k \neq i}}^{n} \sum_{\substack{j' < j \\ j' + d_k > j}}^{T} m_{kj'} r_k \quad \forall i \in \mathcal{I} \quad \forall j \in \mathcal{T} \tag{8}$$



MNLP Model

$$\min F(x) = \sum_{i=1}^{n} cost(x_i)$$

s.t.

$$jm_{ij} + (d_i - 1) \leq T \qquad \forall i \in \mathcal{I} \quad \forall j \in \mathcal{T}$$

$$\sum_{j=1}^{T} m_{ij} \geq 1 \qquad \forall i \in \mathcal{I}$$

$$\sum_{i=1}^{n} m_{ij} r_i \leq R_j \qquad \forall j \in \mathcal{T}$$

$$m_{ij} r_i \leq R - \sum_{\substack{k=0 \\ k \neq i}}^{n} \sum_{\substack{j' < j \\ j' + d_k > j}}^{T} m_{kj'} r_k \qquad \forall i \in \mathcal{I} \quad \forall j \in \mathcal{T}$$

$$m_{ij'} \leq 1 - m_{ij} \qquad \forall i \in \mathcal{I} \quad \forall j' > j \in \mathcal{T}$$

$$x_i \leq m_{ij} T + \epsilon \qquad \forall i \in \mathcal{I} \quad \forall j \in \mathcal{T} \quad \epsilon > 0$$

$$x_i \geq 0 \qquad \forall i \in \mathcal{I}$$

$$m_{ij} \in \{0, 1\} \qquad \forall i \in \mathcal{I} \quad \forall j \in \mathcal{T}$$

NUMERICAL SIMULATIONS

Simulation settings

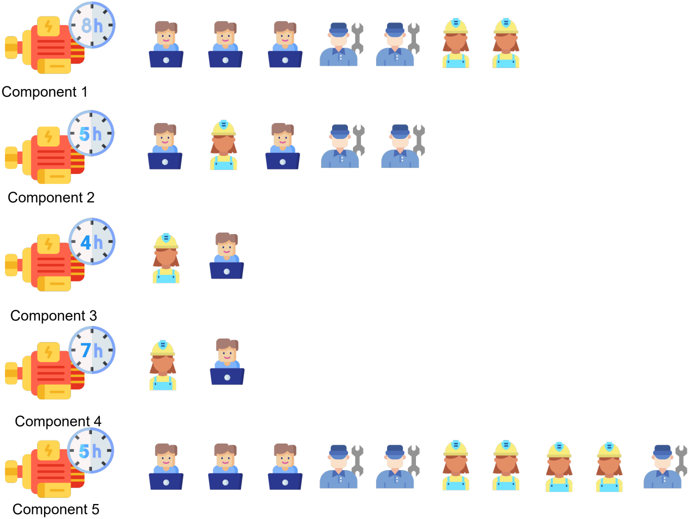
Remark

- Pyomo optimization language
- The Mixed-Integer Nonlinear Decomposition Toolbox (MindtPy) solver

Example 1

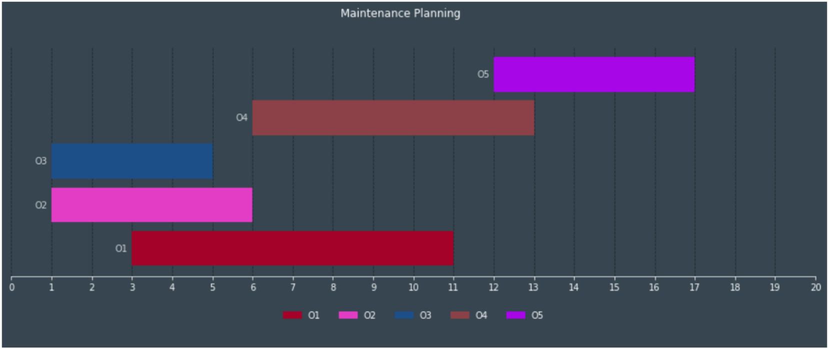
We consider a system with 5 components using the following parameters:

- $T = 20$, $R = 18$



Example 1

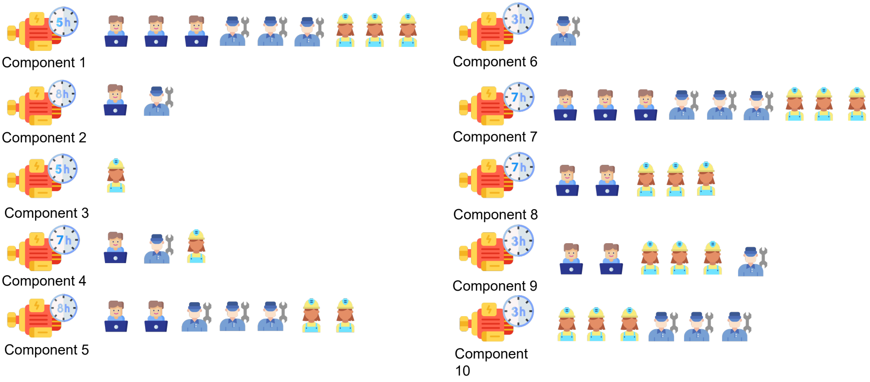
We obtain the following planification



Example 2

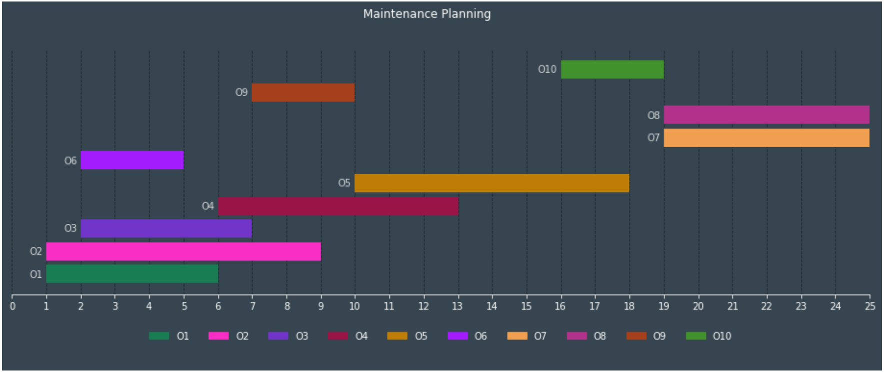
We consider a system with 10 components using the following parameters:

- $T = 25$, $R = 17$



Example 2

We obtain the following result :



Performance Assessment

- Randomly generated systems
- Cost parameters
- Considered metrics:
  1. Execution time
  2. Maintenance Cost
  3. Convergence Rates
  4. Scalability

Performance Assessment

Table: Simulation parameters

Parameters	x_i	t_i	C_i^{Prev}	C_i^{correc}	r_i	d_i	R_j
Values	$[10, 40]$	$[1.25, 2.75]$	$[40, 150]$	$[140, 350]$	$[1, 10]$	$[2, 8]$	$[0, 20]$

Performance Assessment

- Algorithms performance for complex systems with n=5 components

		Planning Horizon						
		$T = 25$	$T = 35$	$T = 45$	$T = 55$	$T = 65$	$T = 75$	$T = 100$
	$C(x)$	**1126.24**	653.28	626.50	**487.51**	494.34	814.06	652.95
MNLP	τ_{system}	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**
	Time (s)	0.628	1.178	1.085	1.829	**2.289**	2.292	3.177

- Algorithms performance for complex systems with n=10 components

		Planning Horizon						
		$T = 25$	$T = 35$	$T = 45$	$T = 55$	$T = 65$	$T = 75$	$T = 100$
	$C(x)$	**1288.19**	900.50	**355.99**	412.182	1272.09	480.73	937.107
MNLP	τ_{system}	100%	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**
	Time (s)	2.091	4.771	4.624	9.012	5.872	**9.027**	11.027

Stochastic Maintenance Optimization of Complex Industrial Systems under Uncertainty of Repair Time and Resources

Stochastic Maintenance Optimization

Problem formulation

- Let O be the set of system's components.
- For each component $o_i \in O$ we associate a duration random variable d_i as Gaussian probabilistic distribution : $\forall o_i \in O, d_i \sim \mathcal{N}(\mu(d_i), \sigma^2(d_i))$
- At each time step j, we assume R_j as a random variable following a Gaussian distribution : $\forall j \in T, R_j \sim \mathcal{N}(\mu(R_j), \sigma^2(R_j))$

Stochastic Maintenance Optimization

Problem formulation

- Uncertainty of the repair time

$$\mathbb{P}\left(jm_{ij} + (d_i - 1) \leq T\right) \geq \alpha_i, \forall i \in \mathcal{I}, \forall j \in \mathcal{T} \tag{9}$$

- Uncertainty of resources availability

$$\mathbb{P}\left(\sum_{i=1}^{n} m_{ij} r_i \leq R_j\right) \geq \beta_i, \forall j \in \mathcal{T} \tag{10}$$

$$\mathbb{P}\left(m_{ij} r_i \leq R_j - \sum_{k=1, k \neq i}^{n} \sum_{\substack{j' < j \\ j' + d_k > j}}^{T} m_{kj'} r_k\right) \geq \gamma_i \forall j \in \mathcal{T} \tag{11}$$

Stochastic Maintenance Optimization

Approximations of Chance Constraints

- Deterministic equivalent formulation of the stochastic repair time constraints :

$$jm_{ij} \leq T + 1 - \mu_i(d) + \phi^{-1}(\alpha_i)\sigma_i(d) \qquad (12)$$

- Deterministic equivalent formulation of the stochastic resources limitation constraints

$$\sum_{i=1}^{n} m_{i,j} \times r_i \leq \mu(R_j) + \phi^{-1}\,(1-\beta)\,\sigma(R_j) \qquad (13)$$

$$m_{ij} r_i - \sum_{k=1, k\neq i}^{n} \sum_{\substack{j'<j \\ j'+d_k>j}}^{T} m_{kj'} r_k \leq \mu(R_j) + \phi^{-1}\,(1-\gamma)\,\sigma(R_j)$$

S Khebbache *et al.*, Stochastic Maintenance Optimization of Complex Industrial Systems under Uncertainty of Repair Time and Resources. The 6^{th} international Conference on System Reliability and Safety. Venise, Italy, Novembre 23-25, 2022.

Deterministic Equivalent Model

$$(DEM): \begin{cases} \min_x \quad C(x) = \sum_{i=1}^{n} cost(x_i) \\ s.t.: \\ jm_{ij} \leq T + 1 - \mu_i(d) + \phi^{-1}(\alpha_i)\sigma_i(d), \forall i \in \mathcal{I} \\ \sum_{i=1}^{n} m_{i,j} \times r_i \leq \mu(R_j) + \phi^{-1}(1-\beta)\,\sigma(R_j); \\ \forall i \in \mathcal{I} \\ m_{ij}r_i - \sum_{k=1,k\neq i}^{n} \sum^{T}_{\substack{j'<j \\ j'+d_k>j}} m_{kj'}r_k \leq \mu(R_j) + \\ +\phi^{-1}(1-\gamma)\,\sigma(R_j) \\ \sum_{x_i=1}^{T} m_{ix_i} \leq \theta\sigma_i + \mu_i, \forall i \in \mathcal{I} \\ x_i \leq m_{i,j} \times T, \forall i \in \mathcal{I} \\ \sum_{j=1}^{T} m_{ij} \geq 1 \quad \forall i \in \mathcal{I} \\ m_{ij'} \leq 1 - m_{ij}, \forall i \in \mathcal{I} \quad \forall j' > j \in \mathcal{T} \\ m_{ij} = \{0,1\}^n, x_i > 0, \forall i \in \mathcal{I} \end{cases} \quad (14)$$

Analysis

$$jm_{ij} \leq T + 1 - \mu_i(d) + \phi^{-1}(\alpha_i)\sigma_i(d), \forall i \in \mathcal{I} \tag{15}$$

For $\sigma(d_i) = 1$, we discuss the following three cases:

- $\alpha = 0$: Then we have $\phi^{-1}(\alpha) = -\infty$. (15) is indicating that no maintenance is possible with confidence levels very low or close to 0.
- $\alpha = 0.5$: Then we have $\phi^{-1}(\alpha) = 0$. (15) is converging totally to the provided original deterministic valid inequality.
- $\alpha = 1$: Then we have $\phi^{-1}(\alpha) = +\infty$. (15) is indicating that maintenance operations are totally relaxed and can be done according to the availability of the repair man.

Analysis

$$\sum_{i=1}^{n} m_{i,j} \times r_i \leq \mu(R_j) + \phi^{-1}(1-\beta)\,\sigma(R_j); \forall j \in \mathcal{T} \tag{16}$$

we propose to assess three cases as follows:

- $\beta = 0$: Then we have $\phi^{-1}(1-\beta) = +\infty$. (16) is indicating that for low values of confidence levels (close to zero), the resources available become theoretically large (infinity).
- $\beta = 0.5$: Then we have $\phi^{-1}(1-\beta) = 0$. (16) is converging to the original and deterministic valid inequality.
- $\beta = 1$: Then we have $\phi^{-1}(1-\beta) = -\infty$. (16) is indicating that maintenance could not occur and this due to lack of resources (repair man).

CONCLUSION & FUTURE WORK

Conclusion & Perspectives

Conclusion

- Considered Resource and Duration Constraints
- A new MNLP model.
- Stochastic formulation to deal with uncertainty

Future Work

- We will extend the result to adress maintenance grouping problems
- We will consider routing constraints for maintenance optimisation in multi-sites factories

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Thank you for your attention

Questions ?