Modèles de percolation pour les réseaux D2D en milieu urbain

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The background: device-to-device (D2D) communications

How does it work? D2D technology consists of direct communication between two users equipments (like smartphones), the signal not needing to go through the nearest base station.

What does it change? Since any user can theoretically play the role of a relay, in cities—where the density of users is very high—operators could reduce their infrastructure costs. Or it could just ease the *uberisation* of the sector.

Question: is a fully functional large-scale D2D network feasible?

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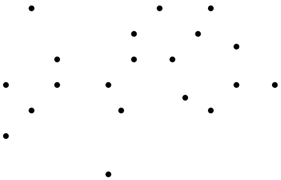

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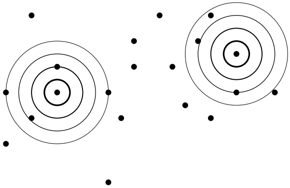

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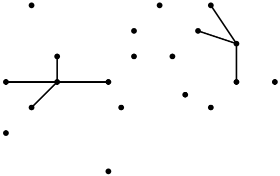

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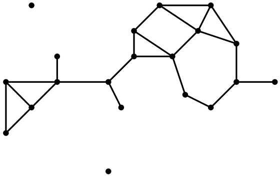

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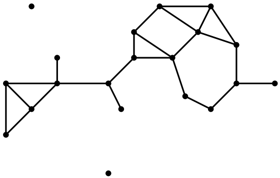

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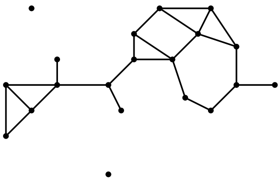

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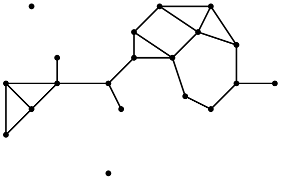

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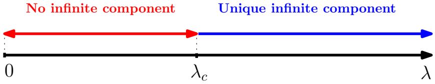

A percolation model for D2D networks in urban areas

Quentin Le Gall and his coauthors recently designed a model to test the scenario of a large-scale D2D network.

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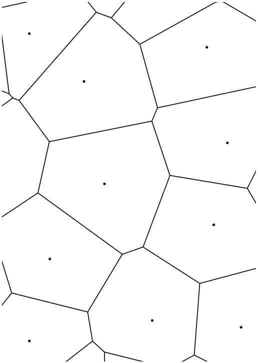

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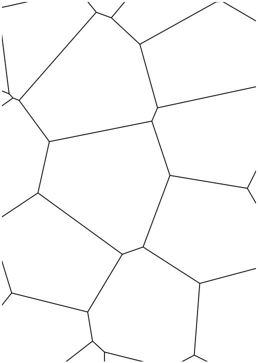

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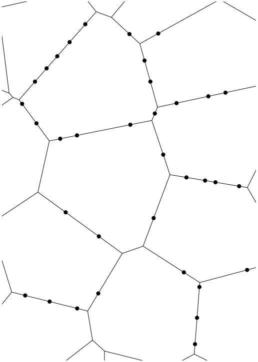

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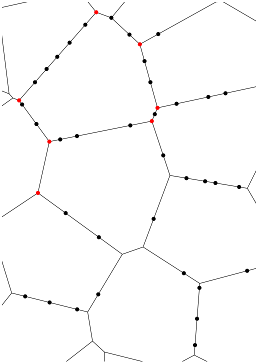

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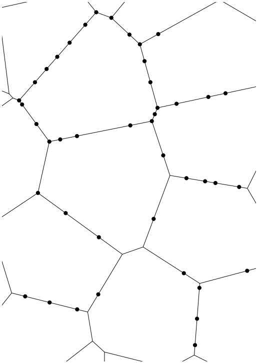

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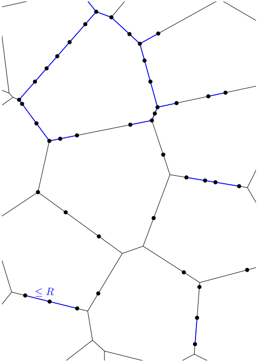

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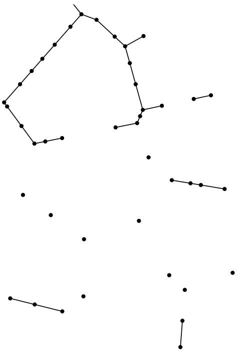

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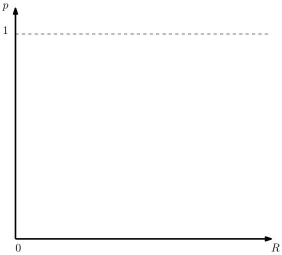

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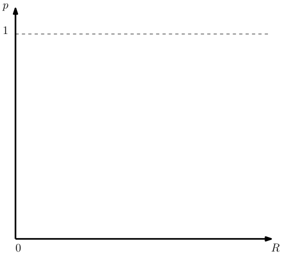

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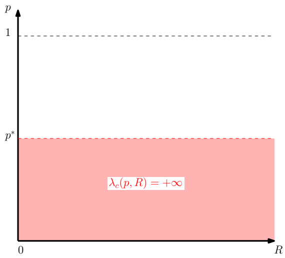

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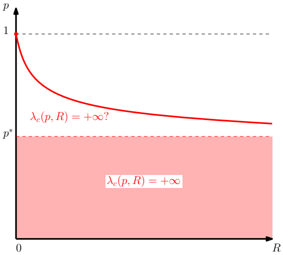

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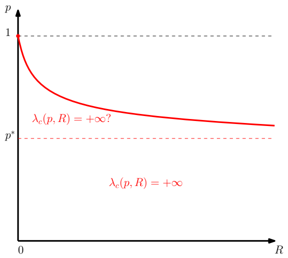

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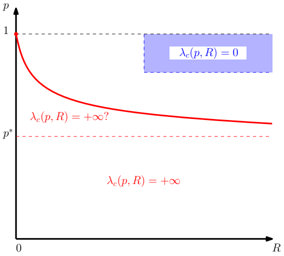

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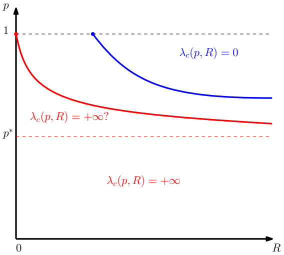

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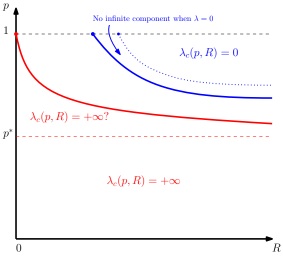

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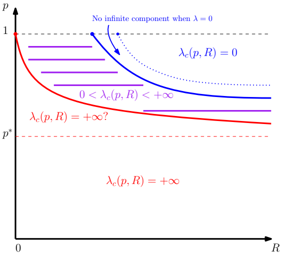

- For $p < p^* \in (0, 1)$, the graph never percolates;
- for p and R large enough, the graph always percolates;
- finally, for some values of p and R, the graph percolates iff the density of users is large enough, i.e. $0 < \lambda_c(p, R) < +\infty$.

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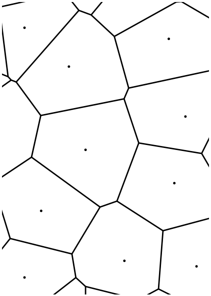

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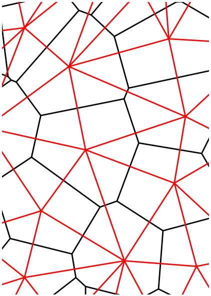

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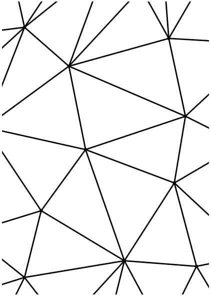

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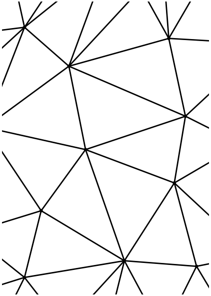
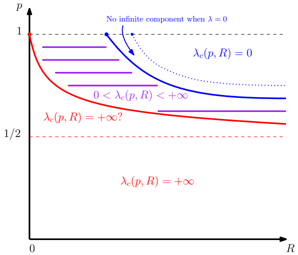


Basically thanks to strong symmetries of the maps.
Site percolation process on it is well understood.

Phase transitions on a Delaunay triangulation



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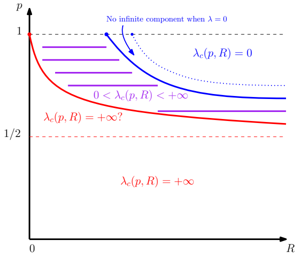

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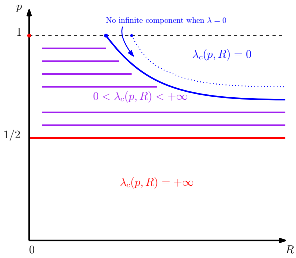

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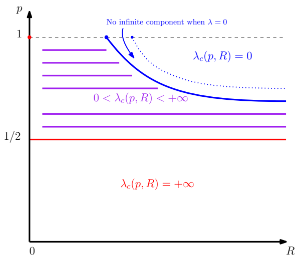

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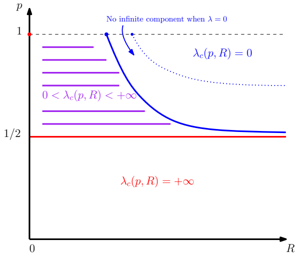

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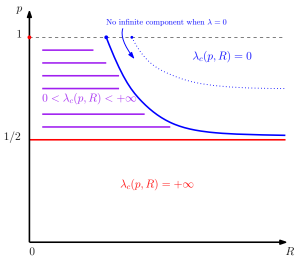

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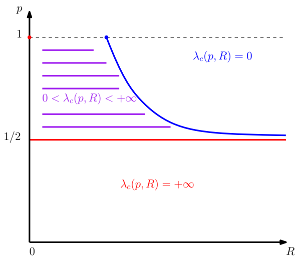

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